

Teacher Guide to Clarification

3.OA.5

Properties of Operations as Strategies to multiply and divide

3.OA.5 Apply **properties of operations** as strategies to multiply and divide.

Examples: If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known. (Commutative property of multiplication.) $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$. (Associative property of multiplication.) Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2) = 40 + 16 = 56$. (Distributive property.)

Properties of Operations

Properties of Operations: Commutative Property, Associative Property, Distributive Property

Commutative property – The word Commutative comes from “commute” or “move around”. When multiplying order does not matter. Student can move numbers around in order to help each individual student multiply mentally. This can be developed through the use of models for basic multiplication.

Example, 3×5 means 3 groups of 5 – I can count 5,10,15

5×3 means 5 groups of 3 – I can count 3,6,9,12,15

Associative property- means to “associate” or “group”. Each student will group numbers in order to multiply mentally and solve the problem efficiently. All different grouping strategies will be shared so students can understand each other’s thinking.

Example, $3 \times 5 \times 2 = ?$

Student 1 $3 \times 5 \times 2 = 30$ or Student 2 $5 \times 2 \times 3 = 30$

- Student 1 thought of the problem as first solving $3 \times 5 = 15$ and then double the answer to make 30.
- Student 2 thought multiplying by 10 would be easier $5 \times 2 = 10$ and then $10 \times 3 = 30$

Distributive property- which means to “distribute” or “break numbers apart”. This is where students can make multiplication problems by breaking numbers into known factors and products. Multiple strategies will be shared and explained.

7 x 6 can be broke apart to make easier problems. 6 can be broke into 5 and 1

7 x 5 = 35 plus one more 7, which is 42.

Coherence and Connections

Below Grade Level	At Grade Level	Above Grade Level
	3.OA.5 3.NBT.3 3.OA.7 3.OA.9 3.MD.7c	4.NBT.5 4.NBT.6

PARCC Evidence Table

Evidence Statement Key	Evidence Statement Text	Clarifications	MP
3.Int.1 EOY	Given a two-step problem situation with the four operations, round the values in the problem, then use the rounded values to produce an approximate solution. See 3.OA.8, 3.NBT.1, 3.NBT.2, 3.NBT.3	i) Be careful that tasks do not require computations beyond the grade 3 expectations. See 4.NBT for computations expected only at the next grade.	4, 6
3.Int.2 EOY	Solve two-step word problems using the four operations requiring a substantial addition, subtraction, or multiplication step, drawing on knowledge and skills articulated in 3.NBT. See 3.OA.8, 3.NBT.2, and 3.NBT.3	i) Addition, subtraction, multiplication and division situations in these problems may involve any of the basic situation types with unknowns in various positions (see CCSS Table 1, p. 88 and Table 2, p. 89 . and the Progression document for Operations and Algebraic Thinking). Substantial (def.) – Values should be towards the higher end of the numbers identified in the standards. ii) If scaffolded, one of the 2 parts must require 2-steps. The other part many consist of 1-step. iii) Conversions should be part of the 2-steps and should not be a step on its own. iv) If the item is 2 points, the item should be a 2 point, unscaffolded item but the rubric should allow for 2-1-0 points.	1, 4

PARCC Mathematics Evidence Tables. (2013, April). Retrieved from:

<http://www.parcconline.org/assessment-blueprints-test-specs>

Students need time to explore these three properties of operations used in order to multiply fluently. The teacher will encourage different ways to think about and solve multiplication problems. Once students use a strategy the teacher can point out the type of strategy used. For example, a student may choose to rearrange three factors; this is an example of the commutative property. This will promote student discovery and empower the student. Students can explore these strategies in a number talk and explain their thinking out loud. Students can also work with a partner or small group to think about multiplication of numbers in different ways.

“Students do not need to know the formal names of the properties but to know the rules that can be applied as strategies in order to multiply and divide.” ~Grade 3 flipbook

Progression Document CC OA K-5 pg. 23-25

Grade 3

Students focus on understanding the meaning and properties of multiplication and division and on finding products of single-digit multiplying and related quotients. 3.OA.1–7 These skills and understandings are crucial; students will rely on them for years to come as they learn to multiply and divide with multi-digit whole number and to add, subtract, multiply and divide with fractions and with decimals.

Note that mastering this material, and reaching fluency in single digit multiplications and related divisions with understanding, 3.OA.7 may be quite time consuming because there are no general strategies for multiplying or dividing all single-digit numbers as there are for addition and subtraction. Instead, there are many patterns and strategies dependent upon specific numbers. So it is imperative that extra time and support be provided if needed.

Common types of multiplication and division situations

Common multiplication and division situations are shown in Table 3. There are three major types, shown as rows of Table 3. The Grade 3 standards focus on Equal Groups and on Arrays. • As with addition and subtraction, each multiplication or division situation involves three quantities, each of which can be the unknown. Because there are two factors and one product in each situation (product _ factor _ factor), each type has one subtype solved by multiplication (Unknown Product) and two unknown factor subtypes solved by division.

• Multiplicative Compare situations are more complex than Equal Groups and Arrays, and must be carefully distinguished from additive Compare problems. Multiplicative comparison first enters the Standards at Grade 4.

Table 3: Multiplication and division situations

	$A \times B = \square$	$A \times \square = C$ and $C \div A = \square$	$\square \times B = C$ and $C \div B = \square$
Equal Groups of Objects	<p>Unknown Product</p> <p>There are A bags with B plums in each bag. How many plums are there in all?</p>	<p>Group Size Unknown</p> <p>If C plums are shared equally into A bags, then how many plums will be in each bag?</p>	<p>Number of Groups Unknown</p> <p>If C plums are to be packed B to a bag, then how many bags are needed?</p>
Arrays of Objects	<p>Unknown Product</p> <p>There are A rows of apples with B apples in each row. How many apples are there?</p>	<p><i>Equal groups language</i></p> <p>Unknown Factor</p> <p>If C apples are arranged into A equal rows, how many apples will be in each row?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into equal rows of B apples, how many rows will there be?</p>
	<p>Unknown Product</p> <p>The apples in the grocery window are in A rows and B columns. How many apples are there?</p>	<p><i>Row and column language</i></p> <p>Unknown Factor</p> <p>If C apples are arranged into an array with A rows, how many columns of apples are there?</p>	<p>Unknown Factor</p> <p>If C apples are arranged into an array with B columns, how many rows are there?</p>
Compare	<p>Larger Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A times as much as the blue hat. How much does the red hat cost?</p>	<p>$A > 1$</p> <p>Smaller Unknown</p> <p>A red hat costs $\\$C$ and that is A times as much as a blue hat costs. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. How many times as much does the red hat cost as the blue hat?</p>
	<p>Smaller Unknown</p> <p>A blue hat costs $\\$B$. A red hat costs A as much as the blue hat. How much does the red hat cost?</p>	<p>$A < 1$</p> <p>Larger Unknown</p> <p>A red hat costs $\\$C$ and that is A of the cost of a blue hat. How much does a blue hat cost?</p>	<p>Multiplier Unknown</p> <p>A red hat costs $\\$C$ and a blue hat costs $\\$B$. What fraction of the cost of the blue hat is the cost of the red hat?</p>

Adapted from box 2–4 of *Mathematics Learning in Early Childhood: Paths Toward Excellence and Equity*, National Research Council, 2009, pp. 32–33.

Notes

Equal groups problems can also be stated in terms of columns, exchanging the order of A and B , so that the same array is described. For example: There are B columns of apples with A apples in each column. How many apples are there?

In the row and column situations (as with their area analogues), number of groups and group size are not distinguished.

In Equal Groups, the roles of the factors differ. One factor is the number of objects in a group (like any quantity in addition and subtraction situations), and the other is a multiplier that indicates the number of groups. So, for example, 4 groups of 3 objects is arranged differently than 3 groups of 4 objects. Thus there are two kinds of division situations depending on which factor is the unknown (the number of objects in each group or the number of groups). In the Array situations, the roles of the factors do not differ. One factor tells the number of rows in the array, and the other factor tells the number of columns in the situation. But rows and columns depend on the orientation of the array. If an array is rotated 90°, the rows become columns and the columns become rows. This is useful for seeing the commutative property for multiplication 3.OA.5 in rectangular arrays and areas. This property can be seen to extend to Equal Group situations when Equal Group situations are related to arrays by arranging each group in a row and putting the groups under each other to form an array. Array situations can be seen as Equal Group situations if each row or column is considered as a group. Relating Equal Group situations to Arrays, and indicating rows or columns within arrays, can help students see that a corner object in an array (or a corner square in an area model) is not double counted: at a given time, it is counted as part of a row or as a part of a column but not both. As noted in Table 3, row and column language can be difficult. The Array problems given in the table are of the simplest form in which a row is a group and Equal Groups language is used (“with 6 apples in each row”). Such problems are a good transition between the Equal Groups and array situations and can support the generalization of the commutative property discussed above. Problems in terms of “rows” and “columns,” e.g., “The apples in the grocery window are

in 3 rows and 6 columns,” are difficult because of the distinction between the number of things *in a row* and the number *of rows*. There are 3 rows but the number of columns (6) tells how many are in each row. There are 6 columns but the number of rows (3) tells how many are in each column. Students do need to be able to use and understand these words, but this understanding can grow over time while students also learn and use the language in the other multiplication and division situations.

Classroom Resource

Refer to the Power Point for examples and fluency building for each of the properties of operations and strategies.

HOT Questions

These questions below are questions that go beyond procedural calculations. These cards or questions can be used as a journal warm up, homework, an assessment, or exit slip. Feel free to use as needed.

3.OA.5 Properties of Operations: Commutative property, Associative property and Distributive property.

<p>1. Draw a picture to show the multiplication of 5×3</p>	<p>2. Write an expression to show 6 groups of 3 children</p>
<p>3. True or False. Explain</p> <p>$3 \times 8 = 8 \times 3$</p>	<p>4. Circle the problem you would prefer to solve and explain why.</p> <p>4×7 or 7×4</p> <hr/>
<p>5. Which two numbers would you multiply first? Why?</p> <p>$4 \times 5 \times 2$</p>	<p>6. Write the number that makes the equation true.</p> <p>$6 \times 9 \times 2 = 9 \times ? \times 2$</p>

7. Break apart 6×7 in order to make the problem easier.

8. My friend broke apart a number below in order to multiply. Are they correct? Yes or No If they are not correct how can you make it correct?

$$7 \times 8 = 4 \times 4 \times 4 \times 4$$

9. Break apart the table below into two multiplication problems.

Write the two multiplication expressions.

Additional Resources

Inside Mathematics Performance Task

<http://www.insidemathematics.org/assets/common-core-math-tasks/boxing%20the%20pots.pdf>

K-5 Teaching Resource – Good center or partner activity

Decomposing a factor

<http://www.k-5mathteachingresources.com/support-files/Decompose-a-Factor.pdf>

Split a factor

<http://www.k-5mathteachingresources.com/support-files/Split-a-Factor.pdf>

Illustrative Mathematics

<https://www.illustrativemathematics.org/illustrations/1821>

Delaware Department of Education – Assessments pg. 14

http://www.doe.k12.de.us/assessment/files/Math_Grade_3.pdf

