**5.NBT.7**

Teacher Guide to Clarification

Instructional Math Materials

**Perform operations with multi-digit whole numbers and with decimals to the hundredths.**

**5.NBT.7** Add, Subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Add, Subtract, Multiply and Divide Decimals with Multiple Strategies**

This standard builds on the work from fourth grade where students are introduced to decimals and compare them. In fifth grade, students begin adding, subtracting, multiplying and dividing decimals. **This work should focus on concrete models and pictorial representations, rather than relying solely on the algorithm.** The use of symbolic notations involves having students record the answers to computations (2.25×3=6.75), but this work should not be done without models or pictures.

**Continue to build students’ conceptual understanding** Students should be able to experience many ways to solve decimal operations. Allow students to find a strategy that works for them. It is important for students to explain their reasoning and understand the reasoning of others

This standard includes students’ reasoning and explanations of how they use models, pictures, and strategies. This standard requires students to extend the models and strategies they developed for whole numbers in grades 1-4 to decimal values.

**Addition**

Students should be able to express that when they add decimals, they add tenths to tenths and hundredths to hundredths. So, when they are adding in a vertical format (numbers beneath each other), it is important that they write numbers with the same place value beneath each other. This understanding can be reinforced by connecting addition of decimals to their understanding of addition of fractions. Adding fractions with denominators of 10 and 100 is a standard in fourth grade.

45.64 + 52.39 =

Students can use their strategies of place value when adding .

45 + 52 = 97

0.60 + 0.30 = 0.90

0.4 + 0.9 = 0.13

97 + 0.90 + 0.13 = 98.03

Students could use a hundreds chart and actually color in the decimal component.

**Student example**

45.64 + 52.39

I can add 45 + 52 and get 97, now I will add the decimals by coloring a hundreds chart.

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|  |  |  |  |  |  |  |  |  | My Answer is 98.03I first added the whole numbers to get 97. Then I realized the two decimals will make a whole number 1 with 0.03 left over so I added 97+1+0.03 |
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**Subtraction Example:** 4−0.3=?

3 tenths subtracted from 4 wholes. The wholes must be divided into tenths.

 4 – 0.3 = ?



The answer is 3 and 7/10 or 3.7

**Subtraction Student examples**

My friend has a gallon of red sports drink. Jessi drank a quarter of it. Angela drank 0.36 of it. How much of the gallon is left over? Give answer in decimal form.

I can subtract 0.25 from 1 and get 0.75 and then Angela drank 0.36 so I have to subtract 0.36 from 0.75. I think there will be less than half of the gallon left.

I can split the 0.36 into 0.25 and 0.11

I know 0.75 – 0.25 = 0.50

Now I just need to subtract 0.50 – 0.11 which is 0.39

There is 0.39 Red Sports Drink left.

What is another way to solve the problem?



**Multiplication Example** An area model can be useful for illustrating products



Students should be able to **describe** the partial products displayed by the area model. For example,

“3/10 times 4/10 is 12/100.

3/10 times 2 is 6/10 or 60/100.

1 group of 4/10 is 4/10 or 40/100.

1 group of 2 is 2.”

**Example of division:** finding the number in each group or share

Students should be encouraged to apply a fair sharing model separating decimal values into equal parts such as:

 

**Example of division:** find the number of groups

Joe has 1.6 meters of rope. He has to cut pieces of rope that are 0.2 meters long. How many can he cut?

To divide to find the number of groups, a student might:

* Draw a segment to represent 1.6 meters. In doing so, s/he would count in tenths to identify the 6 tenths, and be able identify the number of 2 tenths within the 6 tenths. The student can then extend the idea of counting by tenths to divide the one meter into tenths and determine that there are 5 more groups of 2 tenths.



* Count groups of 2 tenths without the use of models or diagrams. Knowing that 1 can be thought of as 10/10, a student might think of 1.6 as 16 tenths. Counting 2 tenths, 4 tenths, 6 tenths, . . . 16 tenths, a student can count 8 groups of 2 tenths.
* Use their understanding of multiplication and think, “8 groups of 2 is 16, so 8 groups of 2/10 is 16/10 or 1 6/10.”

Kansas Association of Teachers of Mathematics (KATM) Flipbooks. Questions or to send feedback: melisa@ksu.edu. Retrieved from: <http://katm.org/wp/wp-content/uploads/flipbooks/5th-Flipbookedited2.pdf>

**Information from the Progression Document NBT on the four decimal operations**

Because of the uniformity of the structure of the base-ten system, students use the same place value understanding for adding and subtracting decimals that they used for adding and subtracting whole numbers.5.NBT.7 Like base-ten units must be added and subtracted, so students need to attend to aligning the corresponding places correctly (this also aligns the decimal points). It can help to put 0s in places so that all numbers show the same number of places to the right of the decimal point. Although whole numbers are not usually written with a decimal point, but that a decimal point with 0s on its right can be inserted (e.g., 16 can also be written as 16*.*0 or 16*.*00). The process of composing and decomposing a base-ten unit is the same for decimals as for whole numbers and the same methods of recording numerical work can be used with decimals as with whole numbers. For example, students can write digits representing new units below on the addition or subtraction line, and they can decompose units wherever needed before subtracting.

General methods used for computing products of whole numbers extend to products of decimals. Because the expectations for decimals are limited to thousandths and expectations for factors are limited to hundredths at this grade level, students will multiply tenths with tenths and tenths with hundredths, but they need not multiply hundredths with hundredths. Before students consider decimal multiplication more generally, they can study the effect of multiplying by 0*.*1 and by 0*.*01 to explain why the product is ten or a hundred times as small as the multiplicand (moves one or two places to the right). They can then extend their reasoning to multipliers that are single-digit multiples of 0*.*1 and 0*.*01 (e.g., 0*.*2 and 0*.*02, etc.).

There are several lines of reasoning that students can use to explain the placement of the decimal point in other products of decimals. Students can think about the product of the smallest base-ten units of each factor. For example, a tenth times a tenth is a hundredth, so 3*.*2 x 7*.*1 will have an entry in the hundredth place. Note, however, that students might place the decimal point incorrectly for 3*.*2x 8*.*5 unless they take into account the 0 in the ones place of 32 x 85. (Or they can think of 0*.*2 x 0*.*5 as 10 hundredths.) They can also think of the decimals as fractions or as whole numbers divided by 10 or 100.5.NF.3 When they place the decimal point in the product, they have to divide by a 10 from each factor or 100 from one factor. For example, to see that 0*.*6 x 0*.*8 = 0*.*48, students can use fractions:

$\frac{6}{10 }$ x $\frac{8}{10}$ = $\frac{48}{100}$ 5.NF.4 Students can also reason that when they carry out the multiplication without the decimal point, they have multiplied each decimal factor by 10 or 100, so they will need to divide by those numbers in the end to get the correct answer. Also, students can use reasoning about the sizes of numbers to determine the placement of the decimal point. For example, 3*.*2 x 8*.*5 should be close to 3 x 9, so 27.2 is a more reasonable product for 3*.*2 x 8*.*5 than 2*.*72 or 272. This estimation-based method is not reliable in all cases, however, especially in cases students will encounter in later grades. For example, it is not easy to decide where to place the decimal point in 0*.*023 x 0*.*0045 based on estimation. Students can summarize the results of their reasoning such as those above as specific numerical patterns and then as one general overall pattern such as “the number of decimal places in the product is the sum of the number of decimal places in each factor.”

General methods used for computing quotients of whole numbers extend to decimals with the additional issue of placing the decimal point in the quotient. As with decimal multiplication, students can first examine the cases of dividing by 0*.*1 and 0*.*01 to see that the quotient becomes 10 times or 100 times as large as the dividend

(see also the Number and Operations—Fractions Progression). For example, students can view 7 ÷ 0*.*1 as asking how many tenths are in 7. Because it takes 10 tenths make 1, it takes 7 times as many tenths to make 7, so 7 ÷ 0*.*1 = 7 x 10 = 70. Or students could note that 7 is 70 tenths, so asking how many tenths are in 7 is the same as asking how many tenths are in 70 tenths, which is 70. In other words, 7 ÷ 0*.*1 is the same as 70 ÷ 1. So dividing by 0*.*1 moves the number 7 one place to the left, the quotient is ten times as big as the dividend. As with decimal multiplication, students can then proceed to more general cases. For example, to calculate 7 x 0*.*2, students can reason that 0*.*2 is 2 tenths and 7 is 70 tenths, so asking how many 2 tenths are in 7 is the same as asking how many 2 tenths are in 70 tenths. In other words, 7 ÷ 0*.*2 is the same as 70 ÷ 2; multiplying both the 7 and the 0*.*2 by 10 results in the same quotient. Or students could calculate 7 ÷ 0*.*2 by viewing 0*.*2 as 2 x 0*.*1, so they can first divide 7 by 2, which is 3*.*5, and then divide that result by 0*.*1, which makes 3*.*5 ten times as large, namely 35. Dividing by a decimal less than 1 results in a quotient larger than the dividend 5.NF.5 and moves the digits of the dividend one place to the left. Students can summarize the results of their reasoning as specific numerical patterns then as one general overall pattern such as “when the decimal point in the divisor is moved to make a whole number, the decimal point in the dividend should be moved the same number of places.”

Common Core Standards Writing Team. (2013, September 19). *Progressions for the Common
 Core State Standards in Mathematics(draft). K-5 Number and Operations in Base 10.* Tucson, AZ: Institute for Mathematics and Educations, University of Arizona.

This section explains how to take the material deeper with decimal operations. Teachers will need to spend time developing this reasoning and understanding vs. just doing the “Decimal Unit”

**Coherence and Connections: Need to Know**

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| **Below Grade Level**  | **At Grade Level** | **Above Grade Level** |
| 4.NBT.4 | **5.NBT.7**5.MD.15.NBT.15.NBT.25.NBT.55.NBT.65.NF.15.NF.45.NF.75.NF.7a5.NF.7b5.NF.7c | 6.NS.3 |

**5.NBT.7** Add, Subtract, multiply and divide decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.

**Examples of Key Advances from Grade 4 to Grade 5**

* In grade 5, students will integrate decimal fractions more fully into the place value system (5.NBT.1–4). By thinking about decimals as sums of multiples of base-ten units, students begin to extend algorithms for multi-digit operations to decimals (5.NBT.7).

*PARCC Model Content Frameworks: Mathematics Grades 3-11 (version 3)*. (2012, November
 1). Retrieved June 3, 2014, from <http://parcconline.org/sites/parcc/files/PARCCMCFMathematicsNovember2012V3_FINAL_0.pdf>

**Take a look at this standard again.**

Not only are students expected to know the 4 operations of decimals but to know the strategies based on place value, properties of operations and relationships between the operations. This is a meaty standard with many grade level connections.

If you are curious as to how students will be assessed, check out the PARCC Evidence Tables below.

**PARCC Evidence Tables**

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| **Evidence****Statement Key** | **Evidence Statement Text** | **Clarifications** | **MP** |
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| 5.C.1-2 **PBA/MYA** | Base explanations/reasoning on the properties of operations. Content Scope: Knowledge and skills articulated in 5.NBT.7  | i) Students need not use technical terms such as *commutative*, *associative*, *distributive*, or *property*. ii) Tasks do not have a context  | MP.3, MP.7, MP.8, MP.6  |

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| MP.3, MP.7, MP.8, MP.6  |

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| 5.NBT.7-1 EOY  |

 | Add two decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  | i) Tasks do not have a context. ii) Only the sum is required; explanations are not assessed here. iii) Prompts may include visual models, but prompts must also present the addends as numbers, and the answer sought is a number, not a picture. iv) Each addend is greater than or equal to 0.01 and less than or equal to 99.99. v) 20% of cases involve a whole number – either the sum is a whole number, or else one of the addends is a whole number presented without a decimal point. (The addends cannot both be whole numbers.)  | 5  |

 | Add two decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  | i) Tasks do not have a context. ii) Only the sum is required; explanations are not assessed here. iii) Prompts may include visual models, but prompts must also present the addends as numbers, and the answer sought is a number, not a picture. iv) Each addend is greater than or equal to 0.01 and less than or equal to 99.99. v) 20% of cases involve a whole number – either the sum is a whole number, or else one of the addends is a whole number presented without a decimal point. (The addends cannot both be whole numbers.)  | MP 5 |
| 5.NBT.7-2  | Subtract two decimals to hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  | i) Tasks do not have a context. ii) Only the difference is required; explanations are not assessed here. iii) Prompts may include visual models, but prompts must also present the subtrahend and minuend as numbers, and the answer sought is a number, not a picture. iv) The subtrahend and minuend are each greater than or equal to 0.01 and less than or equal to 99.99. Positive differences only. (Every included subtraction problem is an unknown-addend problem included in 5.NBT.7-1.) v) 20% of cases involve a whole number – either the difference is a whole number, or the subtrahend is a whole number presented without a decimal point, or the minuend is a whole number presented without a decimal point. (The subtrahend and minuend cannot both be whole numbers.)  | MP 7, MP 5 |
| 5.NBT.7-3  | Multiply tenths with tenths or tenths with hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  | i) Tasks do not have a context. ii) Only the product is required; explanations are not assessed here. iii) Prompts may include visual models, but prompts must also present the factors as numbers, and the answer sought is a number, not a picture. iv) Each factor is greater than or equal to 0.01 and less than or equal to 99.99 v) The product must not have any non-zero digits beyond the thousandths place. (For example, is excluded because the product has an 8 beyond the thousandths place; cf. 5.NBT.3 and see p. 17 of Progression for Number and Operations in Base Ten.) 1.670.340.5678×= vi) Problems are 2-digit 2-digit or 1-digit by 3- or 4-digit. (For example, or .) × 7.85.3× 0.318.24×vii) 20% of cases involve a whole number – either the product is a whole number, or else one factor is a whole number presented without a decimal point. (Both factors cannot both be whole numbers.)  | 7, 5 |
| 5.NBT.7-4  | Divide in problems involving tenths and/or hundredths, using concrete models or drawings and strategies based on place value, properties of operations, and/or the relationship between addition and subtraction; relate the strategy to a written method and explain the reasoning used.  | i) Tasks do not have a context. ii) Only the quotient is required; explanations are not assessed here. iii) Prompts may include visual models, but prompts must also present the dividend and divisor as numbers, and the answer sought is a number, not a picture. iv) Divisors are of the form XY, X0, X, X.Y, 0.XY, 0.X, or 0.0X (cf. 5.NBT.6) where X and Y represent non-zero digits. Dividends are of the form XYZ.W, XY0.X, X00.Y, XY.Z, X0.Y, X.YZ, X.Y, X.0Y, 0.XY, or 0.0X, where X, Y, Z, and W represent non-zero digits. [(Also add XY, X0, and X.)] v) Quotients are either whole #’s or else decimals terminating at the tenths or hundredths place. (Every included division problem is an unknown-factor problem included in 5.NBT.7-3.) vi) 20% of cases involve a whole # – either the quotient is a whole number, or the dividend is a whole number presented without a decimal point, or the divisor is a whole number presented without a decimal point. (If the quotient is a whole number, then neither the divisor nor the dividend can be a whole number.)  | 7, 5  |

*PARCC Mathematics Evidence Tables. (*2013, April). Retrieved from:
<http://www.parcconline.org/assessment-blueprints-test-specs>

PowerPoint used for Discussion

**Classroom Resource**

Below are additional links to number puzzles that can be used as a formative and pre-assessment.

<http://ocs.archchicago.org/Portals/23/CCSS/Fruit%20for%20Thought%201.pdf>

<http://ocs.archchicago.org/Portals/23/CCSS/Fruit%20for%20Thought%202.pdf>

**HOT Questions**

Cut out the number and operation tiles and use them to create the largest possible expression and the smallest possible expression.

Create a class graph to display ever student’s solutions.

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| 4.02 | 8 | 3.23 | .5 | 3.023 | .25 | 2.5 |
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| + | - | / | X | 0 | 1 | .025 |

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| **Number** |  | **Operations** |  | **Number** |
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1. Raymond added these decimal 0.6 + 0.7 and wrote 0.13 as the answer. Why is Raymond incorrect?

2. Insert the decimal point in the product to make the number sentence true.

 2.45 x 6.5 = 15.925

3. Look at the area model below. What is the decimal multiplication number sentence and the product?

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4. Find two decimal numbers to divide that result in a whole number quotient. Explain why your quotient is a whole number. For example, 8.5 ÷ 0.5 = 17

Howard County Wiki <https://grade5commoncoremath.wikispaces.hcpss.org/5.NBT.7>

Inside Mathematics – Problem of the Month<http://www.insidemathematics.org/assets/problems-of-the-month/diminishing%20return.pdf>

<http://www.insidemathematics.org/assets/problems-of-the-month/got%20your%20number.pdf>

K- 5 Teaching ResourcesAdding Decimals with multiple strategies <http://www.k-5mathteachingresources.com/support-files/adding-decimals.pdf>

Subtracting Decimals with multiple strategies
<http://www.k-5mathteachingresources.com/support-files/subtracting-decimals-ver.1.pdf>

Illustrative MathDivision of Decimals<https://www.illustrativemathematics.org/illustrations/292>

Add, Subtract, Multiply and Divide Decimals
<https://www.illustrativemathematics.org/illustrations/1293>

Bridges 1st Edition Grade 5 CCSS Supplement page 195
<http://catalog.mathlearningcenter.org/files/pdfs/SecB5SUPCCSS-B_201309.pdf>